

Global shift symmetry and vacuum energy of matter fields

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We construct the model incorporating both an arbitrary shift of cosmological constant and Goldstone boson corresponding to the spontaneous breaking down the global shift symmetry in the matter action. The gravity breaks down the symmetry explicitly and transforms the Goldstone boson to the inflaton field.

I. GLOBAL SHIFT OF VACUUM ENERGY DENSITY

The cosmological constant of general relativity corresponds to the energy density of vacuum [1, 2]. Let us show that the cosmological constant itself is inherently related to a specific dynamics of matter fields.

Since forces of nature, except the gravity, do not depend on the value of vacuum energy density ρ_Λ , equations of motion for fields of matter are invariant under the global shift

$$\rho_\Lambda = \Lambda_0^4 \mapsto \rho_\Lambda + \Lambda_0^4 u, \quad \partial_\mu u \equiv 0. \quad (1)$$

In this respect the cosmological constant in the action of matter fields looks like an additional global scalar field.

If the action of matter fields is invariant under the global shift with the parameter u in (1), then fixing the vacuum energy means the spontaneous breaking down the global symmetry that leads to the appearance of Goldstone–Nambu boson ϕ with an action invariant under the global shift symmetry

$$\phi \mapsto \phi + f_G u, \quad (2)$$

wherein f_G is the constant of Goldstone boson.

The gravity is not invariant under the global shifts (1) and (2) because the gravitational force depends on the absolute value of energy. Therefore, the gravity breaks down the global invariance of matter action. This breaking leads to generating an effective mass for the boson ϕ , instead of nil potential without the gravity, that makes it the pseudo-Goldstone boson.

Such the program of induced pseudo-Goldstone scalar has been recently considered in [3], wherein we have argued for a non-minimal interaction of ϕ with the curvature scalar and calculated the effective potential in the one-loop approximation for the graviton contribution. A cut-off in loop calculations is related with the Planck mass and the coupling constant of non-minimal interaction, so that after a conformal transformation we arrive to the gravity with the inflaton field consistent with phenomenological parameters of early Universe inflation [4–9] and [10].

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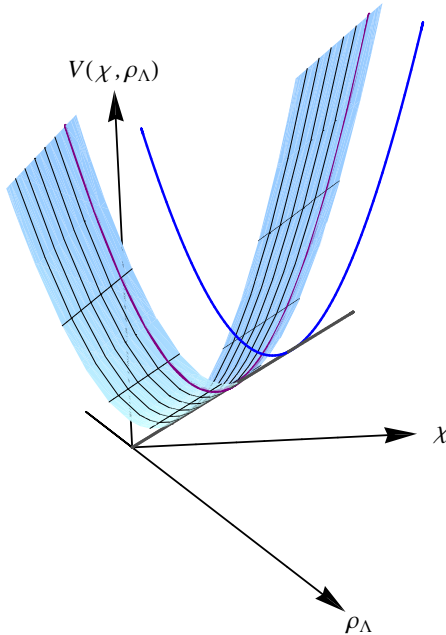


FIG. 1: A potential of field χ versus the density of vacuum energy ρ_Λ .

However, in such the treatment the general assumption of global invariance in the matter field action has been suggested implicitly. The question is whether the action with the global shift of matter fields and global shift of vacuum energy density can be constructed explicitly.

This problem is not trivial, indeed. If the vacuum energy can take an arbitrary value for the matter fields (without the gravity), then we have to expect that a potential of matter fields is not restricted from the bottom. This fact is illustrated in Fig. 1, wherein an ordinary potential of matter field χ depends on the density of vacuum energy ρ_Λ . Then, the instability can occur.

In Section II we present the model possessing the global shift symmetry with the required properties. In Section III we address the problem of stability in two aspects: the limit of zero parameter of instability and stochastic treatment for a source of field that could result in zero tensor of energy-momentum for the stochastically averaged field. Section IV is devoted to short remarks about the Galilean symmetry [11–14] in the model. In Conclusion we summarize the results and discuss its implementations into the theory of inflaton field.

II. MODEL

In the simplest and evident way, the action of two real scalar fields $\psi_{1,2}$

$$S = \int d^4x \mathcal{L}(\psi_1, \psi_2) \quad (3)$$

possesses the global invariance

$$\psi_{1,2} \mapsto \psi_{1,2} + \sqrt{2} f_G u, \quad (4)$$

if the Lagrangian is equal to

$$\mathcal{L}(\psi_1, \psi_2) = \frac{1}{2} \left\{ (\partial_\mu \psi_1)^2 + (\partial_\mu \psi_2)^2 \right\} + \frac{1}{\sqrt{2}} \Lambda_0^3 (\psi_1 - \psi_2). \quad (5)$$

The transformation

$$\phi = \frac{1}{\sqrt{2}} (\psi_1 + \psi_2), \quad \tilde{\phi} = \frac{1}{\sqrt{2}} (\psi_1 - \psi_2), \quad (6)$$

leads to the Lagrangian

$$\mathcal{L}(\phi, \tilde{\phi}) = \frac{1}{2} \left\{ (\partial_\mu \phi)^2 + (\partial_\mu \tilde{\phi})^2 \right\} + \Lambda_0^3 \tilde{\phi}. \quad (7)$$

The solutions of field equations for the vacuum state get the form

$$\langle \phi \rangle = v, \quad \langle \tilde{\phi} \rangle = \tilde{v} + \frac{1}{2} K_{\mu\nu} (x - x_0)^\mu (x - x_0)^\nu, \quad (8)$$

with a symmetric tensor keeping the trace equal to

$$K_\mu^\mu = \Lambda_0^3. \quad (9)$$

At $K_{\mu\nu} x_0^\mu x_0^\nu = 0$ or $x_0 = 0$, the contribution due to $K_{\mu\nu}$ is irrelevant to the problem of constant density of vacuum energy, while it refers to the instability of the model, which will be treated in the next Section. For the sake of simplicity we adopt the condition $x_0 = 0$, although one could easily reformulate all of further statements for $x_0 \neq 0$, of course.

Then, we see that the field ϕ is the Goldstone boson possessing zero mass as well as the nil potential at all, so that the action is invariant under the global shift

$$\phi \mapsto \phi + f_G u, \quad (10)$$

while $\tilde{\phi}$ is responsible for the global shift of cosmological constant: $\rho_\Lambda = -\Lambda_0^3 \tilde{v}$. The global shift of initial fields generates the fractional shifts of vacuum energy density:

$$\psi_{1,2} \mapsto \psi_{1,2} + \sqrt{2} f_G u \quad \Rightarrow \quad (\delta \rho_\Lambda)_{1,2} = \mp \Lambda_0^3 f_G u.$$

The constant terms of expectations

$$\langle \psi_1 \rangle_c = \frac{1}{\sqrt{2}} (v + \tilde{v}), \quad \langle \psi_2 \rangle_c = \frac{1}{\sqrt{2}} (v - \tilde{v}),$$

spontaneously break down the global shift symmetry (4).

Thus, the model explicitly generates the Goldstone boson under the spontaneous breaking down the global shift symmetry.

III. ERASING THE INSTABILITY

Since the potential in (7) is linear in $\tilde{\phi}$, it generates the “accelerated” solution:

$$\tilde{\phi}_A = \frac{1}{2} K_{\mu\nu} x^\mu x^\nu, \quad K_\mu^\mu = \Lambda_0^3. \quad (11)$$

We treat such the solution as the instability, since it appears in the vacuum, too. Let us discuss ways to cancel this instability under the conservation of global symmetry properties and its spontaneous breaking.

A. Limit of zero instability

The simplest method is to take the limit of $\Lambda_0 \rightarrow 0$ at

$$\Lambda_0^3 \tilde{v} = \text{const.}$$

Therefore,

$$\tilde{v} = -\frac{\rho_\Lambda}{\Lambda_0^3} \rightarrow \infty. \quad (12)$$

Thus, the instability is removed, while the cosmological constant could get an arbitrary value ρ_Λ . In this scheme, the Goldstone boson can take an arbitrary expectation value, of course. However, we arrive to the couple massless fields.

B. Stochastic source

Consider the energy-momentum tensor

$$T_{\mu\nu} = \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - g_{\mu\nu} \mathcal{L} \quad (13)$$

for the “accelerated” solution in (11),

$$T_{\mu\nu} \Big|_{\tilde{\phi}_A} = K_{\mu\nu'} x^{\nu'} K_{\nu''\nu''} x^{\nu''} - \frac{1}{2} g_{\mu\nu} \left\{ g^{\mu'\mu''} K_{\mu'\nu'} x^{\nu'} K_{\mu''\nu''} x^{\nu''} + \Lambda_0^3 K_{\mu'\nu'} x^{\mu'} x^{\nu'} \right\}. \quad (14)$$

Since

$$K_\mu^\mu = \Lambda_0^3,$$

the tensor of $K_{\mu\nu}$ represents the source for both the linear potential and solution of field equations. Let us make this source to be stochastic, i.e. the observable quantities are given by average values obtained under the correlations of stochastic source. So, we immediately get

$$\langle K_\mu^\mu \rangle = \Lambda_0^3. \quad (15)$$

Further, in order to keep the Lorentz invariance we have to put

$$\langle K_{\mu\nu} \rangle = \frac{1}{4} g_{\mu\nu} \langle K \rangle, \quad \langle K \rangle = \langle K_\mu^\mu \rangle = \Lambda_0^3, \quad (16)$$

and analogously

$$\langle K_{\mu\nu} K_{\nu'}^\mu \rangle = \frac{1}{4} g_{\nu\nu'} \langle K^2 \rangle, \quad \langle K^2 \rangle = \langle K_{\mu\nu} K^{\mu\nu} \rangle. \quad (17)$$

Next, we fix the correlator

$$\langle K_{\mu\nu} K_{\mu'\nu'} \rangle = A \{ g_{\mu\mu'} g_{\nu\nu'} + g_{\mu\nu'} g_{\nu\mu'} \} + B g_{\mu\nu} g_{\mu'\nu'}, \quad (18)$$

so that

$$5A + B = \frac{1}{4} \langle K^2 \rangle,$$

because

$$g^{\mu\mu'} \langle K_{\mu\nu} K_{\mu'\nu'} \rangle = \frac{1}{4} g_{\nu\nu'} \langle K^2 \rangle.$$

Then, we can calculate the average value of energy-momentum tensor for the “accelerated” solution,

$$\left\langle T_{\mu\nu} \right|_{\tilde{\phi}_A} = g_{\mu\nu} x^2 \left\{ A - \frac{1}{8} \left(\langle K^2 \rangle + \langle K \rangle \Lambda_0^3 \right) \right\} + x_\mu x_\nu (A + B). \quad (19)$$

Tensor (19) becomes relativistically invariant if

$$A + B = 0, \quad (20)$$

hence,

$$A = \frac{1}{16} \langle K^2 \rangle, \quad (21)$$

and

$$\left\langle T_{\mu\nu} \right|_{\tilde{\phi}_A} \Big|_{\text{inv}} = -\frac{1}{16} g_{\mu\nu} x^2 \left\{ \langle K^2 \rangle + 2\Lambda_0^6 \right\}. \quad (22)$$

The translational invariance of average tensor (22) takes place if

$$\langle K^2 \rangle = -2\Lambda_0^6, \quad (23)$$

so that

$$\left\langle T_{\mu\nu} \right|_{\tilde{\phi}_A} \Big|_{\text{inv}} \equiv 0.$$

Thus, for the stochastic source with the correlators

$$\langle K_{\mu\nu} K_{\mu'\nu'} \rangle = -\frac{1}{8} \{ g_{\mu\mu'} g_{\nu\nu'} + g_{\mu\nu'} g_{\nu\mu'} - g_{\mu\nu} g_{\mu'\nu'} \} \Lambda_0^6, \quad \langle K_{\mu\nu} \rangle = \frac{1}{4} g_{\mu\nu} \Lambda_0^3, \quad (24)$$

the instability of “accelerated” solution completely disappears, since the energy-momentum tensor of such the solution is stochastically equal to zero.

Note that the polarization operator

$$\Pi_{\mu\nu,\mu'\nu'} = \frac{1}{2} \{ g_{\mu\mu'} g_{\nu\nu'} + g_{\mu\nu'} g_{\nu\mu'} - g_{\mu\nu} g_{\mu'\nu'} \}$$

corresponds to the projection of symmetric tensor $R^{\mu'\nu'}$ to its Einstein partner,

$$G_{\mu\nu} = \Pi_{\mu\nu,\mu'\nu'} R^{\mu'\nu'} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}.$$

Finally, the shift of “accelerated” solution

$$\tilde{\phi}_A \mapsto \tilde{\phi}_V = \frac{1}{2} K_{\mu\nu} x^\mu x^\nu + \tilde{v}$$

and the stochastic averaging give the vacuum with the energy-momentum tensor equal to

$$\langle T_{\mu\nu} \rangle = -\Lambda_0^3 \tilde{v} g_{\mu\nu},$$

as we have expected.

IV. GALILEAN INVARIANCE

In Minkowski space-time, equations for the motion of both fields ϕ and $\tilde{\phi}$ possess the invariance under the Galilean transformations [11–14]

$$\phi \mapsto \phi + b_\mu x^\mu, \quad \tilde{\phi} \mapsto \tilde{\phi} + \tilde{b}_\mu x^\mu. \quad (25)$$

The stochastic averaging for $\tilde{\phi}$ reduces the Galilean invariance to the global shift of the field, if we consider the energy-momentum tensor, when such the field transformation is equivalent to the introduction of coordinate translation in the form of “accelerated” solution with an appropriate introduction of change in the constant term of the field.

The Galilean invariance of Goldstone boson remains essential especially in the procedure of implementation of invariant interactions of Goldstone boson with itself, matter and gravity [11–14].

V. CONCLUSION

We have just shown that the invariance of non-gravitational forces with respect to the variation of cosmological constant corresponds to the specific dynamical symmetry of global shift in the action of matter fields, while the spontaneous breaking down this global symmetry leads to the appearance of Goldstone boson with the additional Galilean symmetry of its interactions. In order to provide the stable theory we have offered the mechanism of stochastic source, which guarantees the Poincare-invariant value of energy-momentum tensor for the scalar field being under the danger of instability.

The gravitation breaks down the global invariance explicitly, hence, the Goldstone boson acquires an effective potential due to the interaction with the gravity, i.e. due to the graviton loops. The one-loop approximation gives the model of inflaton [3], which is consistent with the modern observations [10]. Moreover, such the origin of inflaton can drive the mechanism for suppressing the cosmological constant [15].

In this respect we have to mention other approaches to both problems of inflaton origin and cosmological constant with various motivations.

So, the pseudo-Goldstone nature of inflaton field is used in *i)* supersymmetric models with flat directions in Kähler potential [16, 17], *ii)* the “natural inflation” with the axion [18], *iii)* the induced gravity with the scale invariance [19].

The theories with a non-metric measure of volume [20–29] operate with the global shift of cosmological constant and give models which are consistent with switching the inflaton potential from the huge plateau in the early Universe into a suppressed plateau at modern times.

Let us comment on the theory of non-metric measure, wherein one makes using the substitution $\sqrt{-\det g_{\mu\nu}} \mapsto \epsilon^{\mu\nu\alpha\beta} \partial_\mu \mathcal{A}_{\nu\alpha\beta}$. So, in the Minkowski limit of $\sqrt{-\det g_{\mu\nu}} \mapsto 1$, we get the action in the form

$$S = \int d^4x \mathcal{L}(\Phi, \partial\Phi) \partial_\mu A^\mu,$$

where Φ is a field of matter, $A^\mu = \epsilon^{\mu\nu\alpha\beta} \mathcal{A}_{\nu\alpha\beta}$ is the field density dual to the initial field of $\mathcal{A}_{\nu\alpha\beta}$. Then, the Lagrange–Euler equations for A^μ read off

$$\partial_\mu L \equiv 0,$$

that means

$$L = \Lambda^4 = \text{const.}$$

Therefore, denoting $\partial_\mu A^\mu = \tilde{\phi}/\Lambda$ we arrive to the action with the linear potential of field $\tilde{\phi}$,

$$S = \int d^4x \Lambda^3 \tilde{\phi},$$

in the form very analogous to our consideration. In [20–29] the field $\tilde{\phi}$ is nontrivially related with the matter and gravitational fields, that constitutes the difference with the simple approach offered in the present paper. By the way, we also note that a non-metric volume measure with density Ψ is equivalent to the introduction of specific interaction with the metric, since $\Psi = U(\Psi, g_{\alpha\beta}) \cdot \sqrt{-\det g_{\mu\nu}}$ at the potential $U = \Psi/\sqrt{-\det g_{\mu\nu}}$. To the current moment, the origin of such the interaction lies beyond any argumentation in the framework of symmetries like the gauge invariance, to our opinion, although the global scale invariance is the favorite of such the motivation mentioned above.

Next, the idea of dynamical evolution is applied to the cosmological constant in [30–36] that transform the problem of cosmological constant to the problem of dark energy.

Thus, we see that the approach offered in the present paper can be used to differentiate complex models of particle physics versus their relevance to the inflaton physics and cosmological constant problem: the most prospective models of matter fields should involve the global invariance spontaneously broken by setting the vacuum expectation values and arbitrary cosmological constant, while the interaction with the gravity should break down the global invariance explicitly.

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